Compressed Sensing MRI

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# Abstract

Table of Contents

Contents

[1.0 Abstract 2](#_Toc477095202)

[2.0 Introduction 4](#_Toc477095203)

[3.0 Results 5](#_Toc477095204)

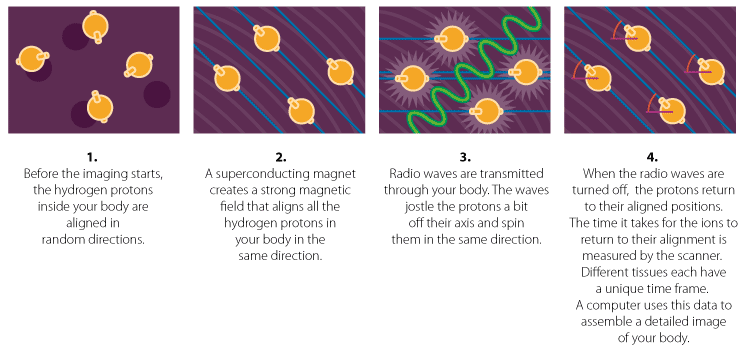
[4.0 Conclusion 6](#_Toc477095205)

# Introduction

2.1 Nuclear Resonance

MRI or (Magnetic Resonant Imaging) is a medical imaging technique which exploits the phenomena known as nuclear magnetic resonance. Nuclear magnetic resonance occurs due to nuclei of atoms possessing an inherent magnetic moment with an associated magnetic spin. These two quantities are dependent on the electron spin and orbital angular momentum of the atom. When a strong static magnetic field (B0)is applied, the nucleus of atoms will polarize and the magnetic moment aligns itself parallel to the static magnetic field. Applying a radio frequency (B1) of a particular frequency will disturb this orientation of magnetic moment and produce a magnetization component transverse to the static field. [1] Switching off this external radio frequency causes the nuclei to return to its externally imposed alignment and emit a detectable radio frequency. The frequency of the return RF signal is proportional to the static field strength.

In MRI, the RF signals are generated by the hydrogen molecules found in the human body. These RF signals are detected by the receiver coils of the MRI machines. A diagram indicating this process can be seen below in figure 1.



Source:<http://www.jwestdesign.com/concept/concept-3.html>

At position r, many different physical properties of tissue proportionally influence the transverse magnetization .One influencing property is proton density however other properties may be emphasized as well. MRI reconstruction aims to visualize depicting the spatial distribution of transverse magnetization.

**2.2 Spatial Encoding and K Space Trajectories**

When the RF signal (B1) is applied, the return RF signal detected by the coils in the MRI machines is the total RF signal to the region of interest where the static magnetic field is applied. For separate RF signals and hence location for protons for the whole image, the protons of the hydrogen atoms can be manipulated through the use of gradient fields. A gradient field is an additional magnetic field in addition with the strong static field(B0) to encode spatial information. By applying an additional magnetic field to a spatial position, the magnetisation of protons in the spatial position will correspond to a precessing frequency and phase. Protons on exactly in the spatial position will vary slightly in frequency depending on the strength of the magnetic field. **This can be shown in the following diagram below.** Through using at least two gradient fields, it is possible to find the location of the protons.

MRI gradient fields vary linearly in space and are signified as , and which correspond to the three Cartesian Axes. Variations in the gradient fields cause location-dependant linear phase dispersion to occur. This allows for the MRI receiver col to detect a linear phase signal dependant on the location. It can be shown that [1] that the signal equation in MRI has the form of a Fourier Integral

Where . In words this equation mean the received signal at time t is the Fourier transform of the object sampled at the spatial frequency

The MRI acquisitions method is based off the Gradient waveforms:

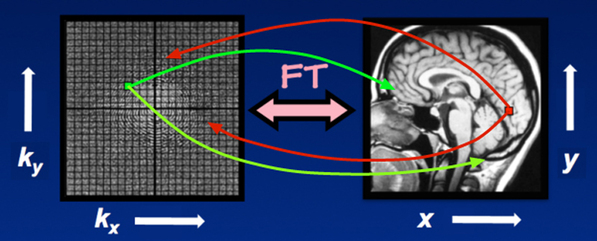
The gradient waveforms with the associated RF pulses used to produce magnetization, are called a pulse sequence [1].

**2.2 Image acquisition and K Space**

The construction of a single MR Image is found through collecting a series of frames of data, called acquisitions. In acquisitions, an RF excitation produced by magnets in the machine produces a new transverse magnetization which is them sampled along a trajectory in k-space.

The k-space is the 2D/ 3D Fourier transform of the MR image measured. It is a grid of raw data of the form ( (phase), (frequency)) obtained directly from the MR signal from MRI machine. Each point in the k space contains phase information and spatial frequency about every pixel in the final image. Conversely, each pixel in the MR image maps to every point in k-space. This concept can be seen below in figure.

This can be seen below in the image



http://mri-q.com/what-is-k-space.html

It should be noted that k-space trajectories/sampling patterns are designed to meet the Nyquist’s criterion which depends on the field of view. Under-sampling in k-space causes aliasing patterns.

Some common k-space trajectories used by MRI machines are shown below:

…

The most common trajectory used by MRIs is the Cartesian Grid using a Cartesian sampling pattern. To get the MR image from Cartesian acquisitions, the inverse Fourier transform is applied to the k-space. For non-Cartesian trajectories different reconstructions such as interpolation schemes (gridding) or back projection.

Using two gradient axes allows for spatial encoding in a 2D plane, known as a single slice of an MR image. For 3D images, multiple slices can me imaged to encode protons in a selected volume.

**Speed of MR scan**

The speed of MRI acquisition and consequently scan time of MRIs are directly correlated to the number of k-space measurements taken by the MRI scan. The speed of the MRI acquisitions by the MRI scan is limited by physical constraints such as slew-rate and maximum amplitude. For high-resolution or wide field of vision images a large number of k-space data is required to satisfy the Nyquist-Shannon criterion. This results in lengthy scan times for patients. The rapid switching and high amplitudes of the gradient fields can also produce peripheral nerve stimulation. This may make patients uncomfortable and involuntarily move. Consequently, motion during the data acquisition results in motion artefacts in the final image which results in image quality degradation.

Motion artefacts come in many forms with the most problematic being motions from cardiac motion, respiratory motion, blood flow and gross body movement. These motions usually occur within a hundred milliseconds to several seconds. These intervals are usually equal or longer than the phase encoding sampling period, hence the majority of motion artefacts come in the phase encoding direction. The most common types of motion artefacts are image blurring and ghosting (misregistration).

Image blurring occurs to random movements which produce a noisy and blurry image. Periodic or ghost images occurs due to periodic movements such as respiration, cardiac beats and arterial or Cerebrospinal fluid pulsations.

https://www.imaios.com/en/e-Courses/e-MRI/Image-quality-and-artifacts/motion

To speed up the existing methods to speed up the MR scan time include accelerating the full k-space acquisition (Echo- Planar Imaging, fast spin echo) and partial acquisition methods of k-space (CS, Parallel imaging, Partial Fourier Imaging)

Full k-space acquisition methods:

**Fast Spin Echo (FSE)**

The conventional method to obtain k-space measurements would be applying a gradient axes to apply a 90 degree pulse and 180 degree pulse. The second pulse is to refocuses spins that have been dephased due to static field homogeneities and produces an echo to be measured by the receiver coils. The time between the center of the first RF pulse and the peak of the spin echo is called the echo time (TE). The sequence repeats itself at the repetition time (TR).

Fast spin echo multiple 180 degree pulses follow each 90 degree pulse at each TR. At each 180 degree pulse a different phase-encoding gradient are is on together compared to the single phase-encoding gradient being turned on once each TR period. This allows for multiple lines in k-space (phase-encoding steps) to be collected within a given TR period. The number of echoes for each TR interval is known as the turbo factor or echo train length (ETL). The number of echoes acquired in a given TR interval is known as the echo train length (ETL) or turbo factor. Typically this ranges from 4-32 for routine imaging but may exceed 200 for rapid imaging/ echo planar techniques. http://mri-q.com/what-is-fsetse.html [source]

FSE offers advantages such as increased SNR (signal noise ratio), reduced susceptibility-induced signal losses and quicker scan times. The major disadvantages of FSE is that it may introduce Gibbs ringing artefacts and image blurring in the phase-encode directions due to the inherent T2 decay during the formation of the echo train. http://mri-q.com/what-is-fsetse.html

A recently discovered method to reduce the number of measurements samples of MRI whilst preserving image quality is to apply compressed sensing techniques to MRI.

**Echo Planar Imaging (EPI)**

EPI involves applying spin-preparation module (which could be a single RF-pulse) , a strong switched frequency-encoding gradient was applied simultaneously with an intermittently "blipped" low-magnitude phase-encoding gradient. This can be seen the diagram below. This results in a zig-zag transversal of the k-space as shown below.

This results in data from a 2D slice being able to collected following a single RF pulse. EPI can acquire slices within 50-100ms and decreases the motion artefacts due to patient motion due to its rapid imaging. A major disadvantage to EPI is its sensitive to inhomogeneity of main magnetic fields. Thus a high performance magnets are required by EPI to avoid gradient errors in imaging.

**Partial Acquisition of k-space techniques**

These methods involve taking less measurements in the k-space to speed up the scan process. The three main methods of partial acquisition include Partial Fourier Sampling, Parallel Imaging and Compressed Sensing.

**Partial Fourier Imaging**

Partial Fourier Imaging exploits inherent property of the Fourier transform known as conjugate (or Hermitian) symmetry. Conjugate symmetry applies to pairs of points (P and Q) located diagonally from each other across the k-space origin. If the data P [a + bi] is a complex, the data at Q is the complex conjugate [a - bi]. This is illustrated in the figure below. Thus in theory, only half the k-space data needs to be collected the other half can be estimated using the conjugate symmetry property. As shown below the major methods of Partial-Fourier imaging are phase-conjugate symmetry and read-conjugate symmetry.

Due to the reduction in k-space measurements, there is up to a reduction in SNR (Signal Noise Ratio) by a factor of compared to the fully-sampled sequence. Source. Furthermore, any phase errors introduced from MR imaging will make symmetrical approximations not perfect.

**Parallel Imaging**

Parallel Imaging is the acquisition of under sampled k-space data from multiple coils that receive data from the same excitation. The multiple coils have localized sensitivities and are not homogenous over the image volume. The localized sensitivities for each coil are defined in well-known arrays known sensitivities maps. When under sampled data is collected, it is done so in a predictable way which the reconstruction method chosen reconstructs a full field-of-view image without aliasing. This is done by under sampling (phase encoding lines) and sampling all (frequencies encoding lines) within these lines. The two most common are Parallel Imaging techniques are SENSE (SENSitivity Encoding) which involves acquiring separate under sampled images from each coil and combining localized sensitivities to unfold the aliased signals mathematically. and GRAPPA (GENERALIZED AUTOCALIBRATING PARTIALLY PARALLEL ACQUISITION. GRAPPA involves acquiring undersampled k space data from coils and acquiring additional data near the centre of the k-space for calibration. This additional k-space data is used to calculate GRAPPA weights and subsequently the missing k-space data before the inverse Fourier Transform.

<https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4459721/figure/F9/>

<https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4459721/>

This reconstruction method can be in the image domain or in the k-space domain.

[**http://www.crcnetbase.com.ezproxy.library.uq.edu.au/doi/pdfplus/10.1201/b19353-6**](http://www.crcnetbase.com.ezproxy.library.uq.edu.au/doi/pdfplus/10.1201/b19353-6)

[**http://www.crcnetbase.com.ezproxy.library.uq.edu.au/doi/pdfplus/10.1201/b19353-6**](http://www.crcnetbase.com.ezproxy.library.uq.edu.au/doi/pdfplus/10.1201/b19353-6)

**Compressed Sensing**

**The final method to speed up the MRI scan time using Partial Acquisition of k-space techniques is applying compressed sensing reconstruction process.**

**Compressed sensing fundamentals:**

**Compressed sensing involves solving the following mathematical problem:**

y = Ax

where:

A is an or sensing matrix which acquires m<<n measurements.

or the measurement matrix

or the N dimensional signal of interest we wish to reconstruct.

And where m << n and is below the Nyquist-Whitter theorem for perfect reconstruction. This mathematical can be seen below in the diagram:

This mathematical equation is an underdetermined linear system with infinite solutions. For unique reconstruction compressed sensing requires three elements must be present: sparsity, sensing matrix and recovery algorithms.

**Sparsity**

For unique reconstruction, the signal is assumed to be sparse. A signal x is k-sparse when it has at most k nonzero or mathematically:

Where , ‖∙‖0 is the 𝑙0 norm. The norms are explained in appendix 1

We let:

Denote the set of all k-sparse signals.

Typically, the signals to be reconstructed are not themselves sparse, but will be sparse in some basis Φ. In this case, x will still be referred to being k-sparse, with the understanding that we can express x as x = Φc where Many sparisfying transforms (Φ) exist such as the Discrete Wavelet transform and Cosine Transform. Both these transforms are used in widely used image formats such as MPEG and JPEG. This thesis will be using the Discrete Wavelet Transform to make the signal sparse.

The Discrete Wavelet Transform is a multiscale representation of the image. Coarse-scale wavelet coefficients represent the low resolution image component and fine-scale wavelet coefficients represent high resolution components. Each wavelet coefficient carries both spatial frequency and position information at the same time. [5] An example of the wavelet transform can be seen below.

http://statweb.stanford.edu/~markad/publications/ddek-chapter1-2011.pdf

Sensing Matrix  
The two major questions involving the design of the sensing matrix (A) is 1. How to design the sensing matrix A to ensure that it preserves the information in the signal x .2 How can we recover the original signal x from measurements y. To ensure unique reconstruction, this section will provide the desirable properties that the matrix A should have for accurate recovery.

Null-space condition

To recover all sparse signals x from y (Ax), then it is immediately clear that for any pair of distinct vectors, we must have , otherwise based of the measurements y, it will be impossible to distinguish from x. Furthermore, if then

with. Thus A uniquely represents all x if and only if the or the nullspace of A contains no vectors in . The nullspace is given as:

Many different ways exist to characterize this property, one of the most common is known as spark.

Definition 1.1 The spark of a given matrix A is the smallest number of columns of A that are linearly dependent denoted as spark (A).

Given an M x N matrix A, if spark(A) ∈ [2, m + 1], this guarantees the following:

Theorem 2.1 For any vector y ∈ , there exists at most one signal x ∈ such that y = Ax if and only if spark(A) > 2k.

When dealing with exactly sparse vectors, the spark provides a complete characterization of when sparse recovery is possible. When dealing with approximately sparse signals a somewhat more restrictive condition of the null-space.

Definition 1.2

A matrix A satisfies the null space property (NSP) of order k if there exists a constant C > 0 such that,

where Λ ⊂ {1, 2, . . . , N} is a subset of indices and = {1, 2, . . . , n}\Λ. is the length n vector obtained by setting the entries of h indexed by to zero. Similarly

To illustrate the implications of the NSP in sparse recovery, we can use the following thereom to measure the performance when dealing with general non-sparse x.

Theorem 2.2 Let represent our specific recovery method and denote a sensing matrix. If (𝑨,Δ) satisfies

Where , then A must necessarily satisfy the NSP of order 2s.

The NSP of order 2s is sufficient to establish a guarantee of the form the previous algorithm to allow for practical recovery a practical recovery algorithm for all possible. s-sparse signals.

**The restricted isometry property**

While the NSP is both sufficient and necessary for establishing of theorem …. These do not account for noise. When the measurements have been corrupted by some error such as quantization errors or noise, stronger conditions must be applied. In Cand`es and Tao[source] an isometry condition was introduced for matrix A.

Definition 2.5:

A matrix A satisfies the restricted isometry property (RIP) of order k if there exists a δk ∈ (0, 1) such that

, (1.7) for all x ∈ .

If a matrix A satisfies the RIP of order 2k, thus A approximately preserves the distance between any pair of k-sparse vectors. This also allows to maintain the stability of solutions and recover a sparse signal from noisy measurements.

To achieve RIP the minimum number of measurements/ measurement bounds are given below in the theorem.

Theorem 4

Let A be an m × n matrix that satisfies the RIP of order 2k with constant δ ∈ (0, ].

Then m ≥ Ck log ()

where C = 1/2 log(√ 24 + 1) ≈ 0.28.

We will see in Sections 1.5 and 1.6 that if a matrix A satisfies the RIP, then this is sufficient for a variety of algorithms to be able to successfully recover a sparse signal from noisy measurements.

<http://statweb.stanford.edu/~markad/publications/ddek-chapter1-2011.pdf>

As proved in source, RIP is strictly stronger than the NSP. The following theorem will prove that if a matrix satisfies RIP it also satisfies NSP.

Theorem 5: Suppose that A satisfies the RIP of order 2k with < − 1. Then A satisfies the NSP of order 2k with constant.

**Mutual coherence:**

While the spark, NSP and RIP provide guarantees for the recovery of k-sparse signals, to verify that any of these are satisfied has computational complexity, since each case must consider submatrices. It is more preferable to use properties of A that provide more concrete guarantees that are also easily computable. The mutual coherence is one such property.

Definition: The mutual coherence of a given M x N matrix A, u(A) is the largest absolute inner product between two columns ,

The lower bound is known as the Welch bound. When n>>m, the lower bound is approximately u() >= 1/sqrt(m). The mutual coherence can be related to the RIP, NSP and spark by employing the Gershgorin circle theorem.

Lemma 2.2 For any matrix A

By combining theorem … with lemma 2.2 we can pose the following contion on A that guarantees uniqueness.

Theorem: if

then for each measurement vector there exists at most one signal  such that

y = Ax.

Theorom 1.7 together with the Welch bound, provides an upper bound that guarantees uniqueness using coherence: . Another method to connect the coherence property to the RIP can be seen below.

Lemma 1.5 If A has unit-norm columns and coherence then A satisfied the RIP of order k with for all

Recovery Algorithm

**5 pages**

CS in MRI

Existing methods half a page

**5 pages**

Image Processing: Image registration

Canny Filter

Affine Transformation

# Results

**2.1 Nuclear Magnetic Resonance**

# Conclusion

References